

Roll No. ....

**337351(14)**

**B. E. (Third Semester)**  
**EXAMINATION, April-May, 2015**  
 (New Scheme)

(Branch : Mech.)

MATHEMATICS—III

Time : Three Hours ] [ Maximum Marks : 80  
 [ Minimum Pass Marks : 28

Note : Part (a) of each question is compulsory. Attempt any two parts from the remaining in each question.

## Unit—I

1. (a) Write Euler's formulae for Fourier series. 2  
 (b) If: 7

$$f(x) = \pi x, \quad 0 \leq x \leq 1$$

$$= \pi(2-x) \quad 1 \leq x \leq 2$$

show that :

$$f(x) = \frac{\pi}{2} - \frac{4}{\pi} \left[ \frac{\cos \pi x}{1^2} + \frac{\cos 3 \pi x}{3^2} + \dots \right]$$

- (c) Obtain the Fourier expansion of  $x \sin x$  as a cosine series in  $(0, \pi)$ . 7

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P. T. O.

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- (d) Expand A in terms of a Fourier series : 7

t	A
0	1.98
T/6	1.30
T/3	1.05
T/2	1.30
2T/3	-0.88
5T/6	-0.25
T	1.98

## Unit—II

2. (a) Write Laplace transform of periodic function. 2  
 (b) Find Laplace transform of the following : 7

(i)  $\int_0^t e^{-t} \cos t \, dt$

(ii)  $\frac{1 - \cos 2t}{t}$

- (c) Find inverse Laplace transform of the following : 7

(i)  $\frac{s}{(s^2 + a^2)^2}$

(ii)  $\log \left( \frac{s+a}{s+b} \right)$

- (d) Solve : 7

$$ty'' + 2y' + ty = \cos t$$

given that :

$$y(0) = 1$$

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## Unit—III

3. (a) Form partial differential equation from : 2

$$z = ax + by + a^2 + b^2$$

- (b) Solve : 7

$$px(z - 2y^2) = (z - qy)(z - y^2 - 2x^3)$$

- (c) Solve : 7

$$(D + D' - 1)(D + 2D' - 3)z = 4 + 3x + 6y$$

- (d) Solve by the method of separation of variables : 7

$$4 \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$$

Given :

$$u = 3e^{-y} - e^{-5y}$$

when  $x = 0$ .

## Unit—IV

4. (a) Write Residue theorem. 2

- (b) If  $f(z)$  is a regular function of  $z$ , prove that : 7

$$\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) |f(z)|^2 = 4 |f'(z)|^2$$

- (c) If: 7

$$\phi(a) = \int_C \frac{3z^2 + 7z + 1}{z - a} dz$$

where  $C$  is the circle  $x^2 + y^2 = 4$ , find the value of the following :

- (i)  $\phi(3)$

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(ii)  $\phi'(1 - i)$

(iii)  $\phi''(1 - i)$

- (d) Apply the calculus of residue, to prove that : 7

$$\int_0^{2\pi} \frac{ad\theta}{a^2 + \sin^2\theta} = \frac{2\pi}{\sqrt{1+a^2}} \quad (a > 0)$$

## Unit—V

5. (a) Define expectation and variance. 2

- (b) Find the moment generating function of :

$$f(x) = \frac{1}{C} e^{-x/c}, 0 \leq x \leq \infty, c > 0$$

Hence find its mean and S. D. 7

- (c) Out of 800 families with 5 children each, how many would you expect to have (i) 3 boys (ii) 5 girls (c) either 2 or 3 boys ? Assume equal probabilities for boys and girls. 7

- (d) Assuming that the diameters of 1000 brass plugs taken consecutively from a machine, form a normal distribution with mean 0.7515 cm. and standard deviation 0.0020 cm. How many of the plugs are likely to be rejected if the approved diameter is  $0.752 \pm 0.004$  cm. ? 7