Roll No. .....

328351(14)

B. E. (Third Semester) EXAMINATION,
April-May, 2015
(New Course)
(Branch : Et & T)
MATHEMATICS–III

Time : Three Hours] [*Maximum Marks : 80*

[*Minimum Pass Marks : 28*

Note : Part (a) is compulsory. Attempt any *two* parts from (b), (c) and (d) of each question.

1. (a) If $L\{f(t)\} = \frac{1}{s(s^2+1)}$, find $\{f(2t)\}$. 2

(b) Find the Laplace transform of the following function : 7

(i) $f(t) = te^{2t} \sin 3t$

(ii) $f(t) = \frac{1 - \cos 3t}{t}$

(c) Use convolution theorem solve : 7

$$y(t) = t + \int_0^t y(\tau) \sin(t - \tau)\, d\tau$$

(d) Use transform method to solve :

$$\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = e^t,$$

given that $x = 2, \frac{dx}{dt} = -1$ at $t = 0$. 7

2. (a) State Cauchy Reimann Equation. 2

(b) If $f(z)$ is an analytic function with constant modulus, show that $f(z)$ is constant. 7

(c) Apply calculus of residue, to prove that : 7

$$\int_0^{\pi} \frac{d\theta}{17 - 8\cos\theta} = \frac{\pi}{15}$$

(d) Find Laurent's expansion 7

$$f(z) = \frac{e^z}{(z-1)^2}$$

about $z = 1$ and deduce that $\int_C f(z)\, dz = 2\pi i\, e$.

3. (a) Write about Karl Pearson's coefficient of correlation. 2

(b) The correlation table given below shows the [illegible] ages of husband and wife of 53 married couples living together on the census night of 1991. Calculate the co-efficient of correlation between the age of the husband and that of the wife. 7

Age of husband	Age of wife						Total
	15-25	25-35	35-45	45-55	55-65	65-75	
15-25	1	1	-	-	-	-	2
25-35	2	12	1	-	-	-	15
35-45	-	4	10	1	-	-	15
45-55	-	-	3	6	1	-	10
55-65	-	-	-	2	4	2	8
65-75	-	-	-	-	1	2	3
Total	3	17	14	9	6	4	53

(c) If θ is the angle between the two regression lines, show that : 7

$$\tan\theta = \frac{1-r^2}{r} \cdot \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}$$

Explain the significance when $r = 0$ and $r = \pm 1$.

(d) Calculate the rank correlation coefficient from the given ahead data showing ranks of 10 students in two subjects : 7

Maths	Physics
3	5
8	9
9	10
2	1
7	8
10	7
4	3
6	4
1	2
5	6

4. (a) If $y(x) = \sum_{m=0}^{\infty} c_m x^{r+m}$ is assumed to be the solution of differential equation :

$$x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 3\left(1+x^2\right)y = 0,$$

then determine the value of r. 7

(b) Obtain the series solution of differential equation. 7

$$\left(2-x^2\right)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 2y = 0$$

(c) Show that : 7

$$J_{\frac{-3}{2}} = \sqrt{\frac{2}{\pi x}}\left[\frac{3-x^2}{x}\sin x - \frac{3}{x}\cos x\right]$$

(d) Prove that :

(i) $P_n'(1) = \frac{n(n+1)}{2}$

(ii) $P_n''(1) = \frac{(-1)^n n(n-1)}{2}$,

where $P_n'(x)$ & $P_n''(x)$ denotes first and second order derivative of $P_n(x)$ with respect to x respectively. 7

(a) From partial differential equation by eliminating arbitrary constant $z = (x+a)(y+a)$. 2

(b) Find the surface satisfying $t = 6x^3y$, where $t = \frac{\partial^2 z}{\partial y^2}$ containing two lines $y = 0, z = 0$ and $y = 1, z = x$. 7

(c) Solve partial differential equation :

$(D - D' - 1)(D - D' - 2)z = e^{2x-y} + xy$ 7

(d) Using method of separation of variable, solve partial differential equation :

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$$

where $u(x, 0) = 6e^{-3x}$.