

324351(14)

B. E. (Third Semester) Examination,

Nov.-Dec. 2015

(New Scheme)

(Elect. Branch)

MATHEMATICS-III

Time Allowed : Three hours

Maximum Marks : 80

Minimum Pass Marks : 28

Note : Attempt all questions. Part (a) from each question is compulsory. Attempt any two parts from (b), (c) and (d) of each question.

1. (a) Let  $f(x)$  be an even function than the value of  $a_n$  is ..... 2

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- (b) Find a half range cosine series for

$$f(x) = \begin{cases} Kx & , 0 \leq x \leq 1/2 \\ K(1-x) & , 1/2 \leq x \leq 1 \end{cases}$$

and deduce the sum of the series

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$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$$

- (c) Find a Fourier series to represent  $x - x^2$  from  $x = -\pi$  to  $x = \pi$ .

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(d) If  $f(x) = \begin{cases} 0 & , -\pi \leq x \leq 0 \\ \sin x & , 0 \leq x \leq \pi \end{cases}$

prove that

$$f(x) = \frac{1}{\pi} + \frac{\sin x}{2} - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\cos 2nx}{4n^2 - 1}$$

Hence show that :

$$\frac{1}{1 \cdot 3} - \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} - \dots \infty = \frac{1}{4}(\pi - 2)$$

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2. (a) Find the Laplace transform of  $\cos^2 2t$ . 2

(b) Find the Laplace transform of : 7

(i)  $L\left(\frac{\cos at - \cos bt}{t}\right)$

(ii)  $L\left\{\int_0^t \frac{e^t \sin t}{t} dt\right\}$

(c) Apply convolution theorem to evaluate : 7

$$L^{-1}\left\{\frac{s^2}{(s^2 + a^2)(s^2 + b^2)}\right\}$$

(d) Use transform method to solve : 7

$$\frac{d^2 x}{dt^2} - 2\frac{dx}{dt} + x = e^t \quad \text{with}$$

$$x = 2, \quad \frac{dx}{dt} = -1 \quad \text{at } t = 0.$$

3. (a) Form the partial differential equation from 2

$$z = f(x - at) + \phi(x + at)$$

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(b) Solve : 7

$$(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$$

(c) Solve : 7

$$(D^2 + 2DD' + D'^2 - 2D - 2D')z = \sin(x + 2y)$$

(d) Using the method of separation of variables, solve : 7

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u \quad \text{where}$$

$$u(x, 0) = 6e^{-3x}$$

4. (a) Define Analytic function. 2

(b) Find the analytic function whose real part is

$$\frac{\sin 2x}{\cosh 2y - \cos 2x} \quad 7$$

(c) Expand region  $f(z) = \frac{1}{(z-1)(z-2)}$  7

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In the given region

(i)  $|z| < 1$

(ii)  $1 < |z| < 2$

(iii)  $|z| > 2$

(d) Evaluate  $\int_C \frac{z^2 - z + 1}{z - 1} dz$  where  $C$  is the circle 7

(i)  $|z| = 1$

(ii)  $|z| = 1/2$

5. (a) Write Damping rule of z-transform. 2

(b) Find the z-transform of : 7

(i)  $3n - 4 \sin \frac{n\pi}{2} + 5a$

(ii)  $\cos \left( \frac{n\pi}{2} + \frac{\pi}{4} \right)$

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(c) Use convolution to evaluate : 7

$$z^{-1} \left\{ \frac{z^2}{(z-a)(z-b)} \right\}$$

(d) Using the residue method find the inverse z-transform of : 7

$$\frac{10z}{(z-1)(z-2)}$$

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