

**322311****(14)**

BE (3<sup>rd</sup> Semester)  
Examination, Nov.-Dec. 2013

Branch : CSE, IT

**MATHEMATICS - III**

Time Allowed : Three Hours

Maximum Marks : 80

Minimum Pass Marks : 28

Note : In each question answer part (a) is compulsory and any two parts from remaining. Part (a) is of 2 marks and remaining questions are of 7 marks each.

Q. 1. (a) Explain Dirichlet conditions for  $f(x)$  to be expanded in Fourier series. 2

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(b)  $f(x) = x + x^2$  for  $-\pi < x < \pi$  and  $f(x) = \pi^2$  for  $x = \pm \pi$ . Explain  $f(x)$  in Fourier series and

show that :

$$x + x^2 = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} (-1)^n \left\{ \frac{4}{n^2} \cos nx - \frac{2}{n} \sin nx \right\}$$

(c) Obtain Fourier series for the function  $f(x)$

given by :

$$f(x) = \begin{cases} 1 + \frac{2x}{\pi} & -\pi \leq x < 0 \\ 1 - \frac{2x}{\pi} & 0 \leq x < \pi \end{cases}$$

Deduce that  $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$

(d) The following table gives the variations of periodic current over a period.

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t	0	T/6	T/3	T/2	2T/3	5T/6	T
Amp	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

Show that there is a direct current part is

0.75 amp in variable current and obtain the

amplitude of the first harmonic. 7

Q. 2. (a) Write down the mathematical expression for

Laplace transform of periodic function. 2

(b) Find the Laplace transform of : 7

(i)  $t e^{-t} \sin 3t$

(ii)  $\frac{e^{-at} - e^{-bt}}{t}$

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(c) Solve the differential equation using Laplace

transform : 7

$$\frac{d^2x}{dt^2} + 9x = \cos 2t, \text{ if } x(0) = 1$$

$$x\left(\frac{\pi}{2}\right) = -1$$

(d) (i) Apply convolution theorem to evaluate : 3½

$$L^{-1} \frac{s}{(s^2 + a^2)^2}$$

(ii) Find the inverse of the transform : 3½

$$L^{-1} \left[ \frac{1}{2} \log \frac{(s^2 + b^2)}{(s^2 + a^2)} \right]$$

Q. 3. (a) Write down the mathematical expression for

Cauchy integral formula. 2

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(5)

(b) If  $f(z)$  is a regular function of  $z$ , prove that : 7

$$\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) |f(z)|^2 = 4|f'(z)|^2$$

(c) Obtain Laurent's expansion for the function

$$f(z) = \frac{1}{z^2 \sin hz} \text{ and evaluate } \int_c \frac{dz}{z^2 \sin hz}$$

where  $c$  is the circle  $|z - 1| = 2$  7

(d) Apply the calculus of residue to prove that : 7

$$\int_0^{2\pi} \frac{d\theta}{1 - 2p \sin \theta + p^2} = \frac{2\pi}{1 - p^2} \quad (0 < p < 1)$$

Q. 4. (a) Form the partial differential equation

from : 2

$$z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$$

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(b) Solve the following equation :

$$(z^2 - 2yz - y^2) p + (xy + zx) q = xy - zx$$

(c) Solve the following differential equation : 7

$$(D^3 + D^2D' - DD'^2 - D'^3)z = e^x \cos 2y$$

(d) Solve the following equation by the method of separation of variables : 7

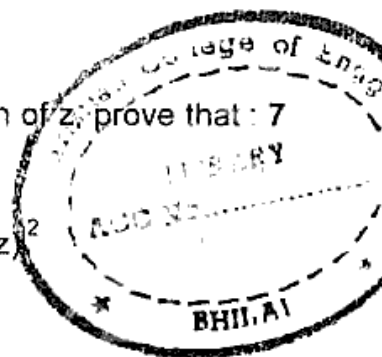
$$4 \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u, \text{ given } u = 3e^{-y} - e^{-5y} \text{ when}$$

$$z = 0.$$

Q. 5. (a) Define expectation of probability distribution of

a variate. 2

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(7)

- (b) Find the moment generating function of the exponential distribution :

$$f(x) = \frac{1}{c} e^{-x/c}, \quad 0 < x \leq \infty, c > 0$$

Hence find its mean and S.D.

- (c) If on an average 1 vessel in every 10 is wrecked, find the probability that out of 5 vessel expected to arrive, at least 4 will arrive safely. 7

- (d) Assuming that the diameters of 1000 brass plugs taken consecutively from a machine, form a normal distribution with mean

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0.7515 cm, and standard deviation 0.0020 cm

how many of the plugs are likely to be

rejected if the approved diameter is  $0.752 \pm$

0.004 cm ? 7

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3,730