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322311 (14)

BE (3^{rd} Semester)

Examination, Nov.-Dec., 2013

Branch : CSE, IT

MATHEMATICS - III

Time Allowed : Three Hours

Maximum Marks : 80

Minimum Pass Marks : 28

Note : In each question answer part (a) is compulsory and any two parts from remaining. Part (a) is of 2 marks and remaining questions are of 7 marks each.

Q. 1. (a) Explain Dirichlet conditions for f(x) to be expanded in Fourier series. 2

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(b) $f(x) = x + x^2$ for $-\pi < x < \pi$ and $f(x) = \pi^2$ for $x = \pm\pi$. Explain f(x) in Fourier series and show that : 7

$$x + x^2 = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} (-1)^n \left\{\frac{4}{n^2} \cos nx - \frac{2}{n} \sin nx\right\}$$

(c) Obtain Fourier series for the function f(x) given by : 7

$$f(x) = \begin{cases} 1 + \frac{2x}{\pi} & -\pi \le x < 0 \\ 1 - \frac{2x}{\pi} & 0 \le x < \pi \end{cases}$$

Deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} ..... = \frac{\pi^2}{8}$

(d) The following table gives the variations of periodic current over a period.

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t	0	$T/6$	$T/3$	$T/2$	$2T/3$	$5T/6$	T
Amp	1.98	1.30	1.05	1.30	−0.88	−0.25	1.98

Show that there is a direct current part is 0.75 amp in variable current and obtain the amplitude of the first harmonic. **7**

Q. 2. (a) Write down the mathematical expression for Laplace transform of periodic function. **2**

(b) Find the Laplace transform of : **7**

(i) $t\, e^{-t} \sin 3t$

(ii) $\dfrac{e^{-at} - e^{-bt}}{t}$

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(c) Solve the differential equation using Laplace transform : **7**

$$\frac{d^2x}{dt^2} + 9x = \cos 2t, \quad \text{if } x(0) = 1$$

$$x\left(\pi/2\right) = -1$$

(d) (i) Apply convolution theorem to evaluate : **3½**

$$L^{-1} \frac{s}{(s^2 + a^2)^2}$$

(ii) Find the inverse of the transform : **3½**

$$L^{-1}\left[\frac{1}{2}\log\frac{(s^2 + b^2)}{(s^2 + a^2)}\right]$$

Q. 3. (a) Write down the mathematical expression for Cauchy integral formula. **2**

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(b) If f(z) is a regular function of z, prove that : 7

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$$

(c) Obtain Laurent's expansion for the function

$$f(z) = \frac{1}{z^2 \sinh z} \text{ and evaluate } \int_c \frac{dz}{z^2 \sinh z};$$

where c is the circle $|z - 1| = 2$ 7

(d) Apply the calculus of residue to prove that : 7

$$\int_0^{2\pi} \frac{d\theta}{1-2p\sin\theta+p^2} = \frac{2\pi}{1-p^2} (0<p<1)$$

Q. 4. (a) Form the partial differential equation from : 2

$$z = y^2 + 2f\left(\frac{1}{x}+\log y\right)$$

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(b) Solve the following equation :

$$(z^2 - 2yz - y^2)\,p + (xy + zx)\,q = xy - zx$$

(c) Solve the following differential equation : 7

$$(D^3 + D^2D' - DD'^2 - D'^3)z = e^x.\cos 2y$$

(d) Solve the following equation by the method of separation of variables : 7

$$4\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=3u, \text{ given } u = 3e^{-y} - e^{-5y} \text{ when}$$

$z = 0$.

Q. 5. (a) Define expection of probability distribution of a variate. 2

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(b) Find the moment generating function of the exponential distribution :

$$f(x) = \frac{1}{c}e^{-x/c}, \quad 0 < x \leq \infty, c > 0$$

Hence find its mean and S.D. 7

(c) If on an average 1 vessel in every 10 is wrecked, find the probability that out of 5 vessel expected to arrive, at least 4 will arrive safely. 7

(d) Assuming that the diameters of 1000 brass plugs taken consecutively from a machine, form a normal distribution with mean

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0.7515 cm, and standard deviation 0.0020 cm how many of the plugs are likely to be rejected if the approved diameter is 0.752 ± 0.004 cm ? 7

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