Roll No. ....

320351(14)

BE (3rd Semester)

Examination, April-May, 2018

(New Scheme)

Mathematics-III

Time Allowed : 3 hours

Maximum Marks : 80

Minimum Pass Marks : 28

Note : (*i*) Part (*a*) of each question is compulsory. Attempt any **two** parts from (*b*), (*c*) and (*d*) of each question. Area under the normal curve table is allowed.

(*ii*) The figures in the right-hand margin indicate marks.

1\. (*a*) If $x = c$ is a point of discontinuity, then the Fourier series of $f(x)$ at $x = c$ gives $f(x)$ = ___ (Fill in the blank) [2]

(*b*) If $f(x) = |\cos x|$, then expand $f(x)$ as a Fourier series in the interval $(-\pi, \pi)$. [7]

(*c*) Expand $f(x)$ as the Fourier series of sine terms, where

$$f(x) = \begin{cases} \frac{1}{4} - x, & \text{if} \quad 0 < x < \frac{1}{2} \\ x - \frac{3}{4}, & \text{if} \quad \frac{1}{2} < x < 1 \end{cases}$$

[7]

(*d*) Obtain the first three coefficients in the Fourier cosine series for y, where y is given in the following table : [7]

x :	0	1	2	3	4	5
y :	4	8	15	7	6	2

2\. (*a*) Define unit-step function and write its Laplace transform. [2]

(*b*) If $f(t)$ is a periodic function with period T, i.e., $f(t+T) = f(t)$, then prove that

$$L\{f(t)\} = \frac{\int_0^T e^{-st} f(t)\, dt}{1 - e^{-sT}}$$

[7]

(*c*) Find the inverse Laplace transforms of the following :

(*i*) $\dfrac{5s+3}{(s-1)(s^2+2s+5)}$ [4]

(*ii*) $\cot^{-1}\left(\dfrac{s}{2}\right)$ [3]

(*d*) Solve

$$ty'' + 2y' + ty = \cos t$$

given that $y(0)=1$. [7]

3\. (*a*) Form the partial differential equation by eliminating the arbitrary function from $z = e^{ay}\phi(x-y)$. [2]

(*b*) Solve : [7]

$$x^2(y-z)p + y^2(z-x)q = z^2(x-y)$$

(c) Solve : [7]

$$(D^2 + 2DD' + D'^2 - 2D - 2D')z = \sin(x + 2y)$$

where $D = \dfrac{\partial}{\partial x}$ and $D' = \dfrac{\partial}{\partial y}$.

(d) Using the method of separation of variables,

solve $4\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} = 3u$, given that

$u(0, y) = 3e^{-y} - e^{-5y}$ [7]

4. (a) Evaluate $\displaystyle\int_C \frac{z^2 - z + 1}{z - 1}\,dz$, where C is the circle $|z| = \dfrac{1}{2}$. [2]

(b) Show that polar forms of Cauchy-Riemann equations are

$$\frac{\partial u}{\partial r} = \frac{1}{r}\cdot\frac{\partial v}{\partial \theta} \quad \text{and} \quad \frac{\partial v}{\partial r} = \frac{1}{r}\cdot\frac{\partial u}{\partial \theta}$$

Hence deduce that

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial u}{\partial r} + \frac{1}{r^2}\cdot\frac{\partial^2 u}{\partial \theta^2} = 0$$ [7]

(c) Find the Laurent's expansion of

$$f(z) = \frac{7z - 2}{(z+1)z(z-2)}$$

in the region $1 < z + 1 < 3$. [7]

(d) Show that

$$\int_0^{2\pi} \frac{\cos 2\theta\, d\theta}{1 - 2a\cos\theta + a^2} = \frac{2\pi a^2}{1 - a^2}$$

where $a^2 < 1$. [7]

5. (a) What are the mean and standard deviation of binomial distribution? [2]

(b) The probability density $p(x)$ of a continuous random variable is given by

$$p(x) = y_0 e^{-|x|}, \quad -\infty < x < \infty$$

Prove that $y_0 = \dfrac{1}{2}$. Find the mean and variance of the distribution. [7]

(c) If the probability of a bad reaction from a certain injection is 0.001, determine the chance that out of 2000 individuals more than two will get a bad reaction. [7]

(d) X is a normal variate with mean 30 and S.D. 5. Find the probabilities that

(i) $26 \le X \le 40$, (ii) $X \ge 45$ and

(iii) $|X - 30| > 5$. [7]